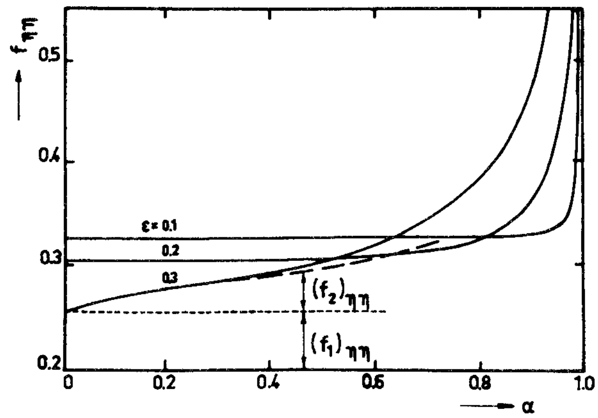

Fig. 4 Wall shear stress profiles for the continuously moving surface for various $\varepsilon = u_w/U$ after Ref. 4; ---similarity solution.

The transformation eliminating the singularity at the trailing edge is

$$\bar{\eta} = \bar{y}(u_w/v \cdot \bar{x})^{1/2}, \quad (\bar{f}_2)_{\bar{\eta}} = (u/-u_w) \tag{10}$$

where the coordinates $\bar{x}$, $\bar{y}$ are fixed to the trailing edge. With $\bar{x} \to 0$ the transformed momentum equation becomes

$$(\bar{f}_2)_{\bar{\eta}\bar{\eta}\bar{\eta}} + \tfrac{1}{2}\bar{f}_2(\bar{f}_2)_{\bar{\eta}\bar{\eta}} = 0 \tag{11}$$

The boundary conditions are at the wall

$$(\bar{f}_2)_{\bar{\eta}} = 1, \quad \bar{f}_2 = 0 \tag{12.1}$$

and at the outer edge of the boundary layer

$$(\bar{f}_2)_{\bar{\eta}} = 0 \tag{12.2}$$

Equation (11) with boundary conditions (12.1) and (12.2) applies to the region of reverse flow which, with $\bar{x} \to 0$, becomes independent of the forward flow.

In Fig. 4, the dimensionless wall shear stress as given in Ref. 4 is shown as function of $\alpha = x/L$ for different values of $\varepsilon = u_w/U$, where L is the length of the moving surface. Using transformation (8), the problem numerically was solved in the interaction region $0 < \alpha < 1$. Since this transformation removes the singularity at the leading edge only, the second singularity at the trailing edge $\alpha = 1$ made problems for the numerical solution affecting its accuracy near $\alpha = 1$, as pointed out in Ref. 4.

The asymptotic value of the wall shear stress at the trailing edge can now be taken from the solution of Eq. (11) which is $(\bar{f}_2)_{\bar{\eta}\bar{\eta}} = 0.444$, as in the foregoing example of the shock induced boundary layer. Transformed to the coordinates being fixed to the leading edge, the asymptotic value of the wall shear stress shown in Fig. 4 with $\alpha \to 1$ is

$$f_{\eta\eta} = (f_2)_{\eta\eta} = 0{,}444\,\varepsilon^2 \cdot (\alpha/1-\alpha)^{1/2}$$

It is interesting to note that for stronger reverse flow ($\varepsilon = 0.3$) although the problem is nonlinear, linear superposition of the wall shear stresses, $(f_1)_{\eta\eta}$ and $(f_2)_{\eta\eta}$, near the leading edge, yields agreement with the numerical solution, as can be seen from Fig. 4.

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Taylor-Görtler Instability of a Boundary Layer with Suction or Blowing

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Introduction

THE purpose of this Note is to consider theoretically how the suction or blowing from a slightly concave permeable wall will affect the instability of the incompressible two-dimensional laminar boundary layer to the onset of longitudinal vortices. As is well known, the critical Reynolds number for Tollmien-Schlichting waves depends strongly on the basic profile of the streamwise velocity component in the laminar boundary layer, while the critical Görtler parameter for the longitudinal vortices varies insensibly with a large change of the streamwise velocity profile.[1,2] We are here interested in a role of the normal velocity component in the Taylor-Görtler instability. In particular, suction or blowing from a permeable wall induces a notable normal velocity component in the boundary layer.

Analysis

A freestream with a uniform velocity U_∞ is directed along the axis x, where x is the arc length of the basic concave streamlines, y is the distance measured perpendicular to the wall and z normal to the two axes, (u, v, w) are the corresponding components of the velocity vector. The radius R of the curvature on the wall remains constant in the x-direction and is far larger than the momentum thickness θ of the boundary layer. The normal velocity profile v_0 shows in general a wide variety in dependence on a manner of suction or blowing. We put the suction distribution $v_0 = -C(vU_\infty/x)^{1/2}/2$ along the surface, where v denotes the kinematic viscosity of the fluid. In this case the boundary-layer flow has a similar solution.[3] We shall now suppose that the basic laminar boundary-layer flow is slightly perturbed with the type of longitudinal vortices, which may be expressed as

$$u = u_0(x, y) + u_1(y)\cos\alpha z \exp(\textstyle\int \beta\, dx) \tag{1a}$$

$$v = v_0(x, y) + v_1(y)\cos\alpha z \exp(\textstyle\int \beta\, dx) \tag{1b}$$

$$w = w_1(y)\sin\alpha z \exp(\textstyle\int \beta\, dx) \tag{1c}$$

with the wavenumber α and a measure β of the rate of growth of the disturbances. We have finally a system of perturbation equations governing the present linear instability problem in the neutral state ($\beta = 0$) as follows:

$$(D^2 - \bar{v}_0 D + D\bar{v}_0 - \sigma^2)\bar{u}_1 = (D\bar{u}_0)\bar{v}_1 \tag{2}$$

$$(D^2 - \bar{v}_0 D - D\bar{v}_0 - \sigma^2)(D^2 - \sigma^2)\bar{v}_1 = -2\sigma^2 G^2 \bar{u}_0 \bar{u}_1 \tag{3}$$

where $\bar{u}_0 = u_0/U_\infty$, $\bar{v}_0 = v_0\theta/v$, $\bar{u}_1 = u_1/U_\infty$, $\bar{v}_1 = v_1\theta/v$, $\sigma = \alpha\theta$, $G = (U_\infty\theta/v)(\theta/R)^{1/2}$, $\eta = y/\theta$ and $D = d/d\eta$. G is called Görtler parameter. Equations (2) and (3) for $\bar{v}_0 = 0$ coincide with the classical equations.[1,2]

We assume no slip condition at the permeable surface ($\eta = 0$). This assumption may be approximately valid when holes or slits of the permeable wall are fine enough. The boundary conditions then become $\bar{u}_1 = \bar{v}_1 = D\bar{v}_1 = 0$ in consideration of the equation of continuity. It may be reasonable to take the other boundary condition at a point ($\eta \to \infty$) far enough from the wall, because the longitudinal vortices will extend over the outer edge of the boundary layer. The boundary condition under which all perturbations vanish at the infinity ($\eta \to \infty$) leads again to $\bar{u}_1 = \bar{v}_1 = D\bar{v}_1 = 0$. It is, however, necessary in numerical integrations to give a finite value (η_m) instead of the infinity. The value η_m should be taken greater as

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the dimensionless wavenumber σ of the longitudinal vortices becomes smaller. It might be expected from the Görtler's calculation[1] that a too small value of η_m results in an overestimation of the Görtler parameter G in the state of neutral stability. Instead of the outer boundary condition for $\eta \to \infty$, we use the three following identities for the outside of the boundary layer ($\eta \geqq \eta_e$), which are obtained from Eqs. (2) and (3) with $\bar{u}_0 = 1.0$ and $D\bar{u}_0 = 0$

$$D\bar{u}_1 + \lambda_1 \bar{u}_1 = 0 \tag{4}$$

$$D^2\bar{v}_1 + (\lambda_1 + \sigma)D\bar{v}_1 + \sigma\lambda_1\bar{v}_1 = k(\sigma - \lambda_1)\bar{u}_1 \tag{5}$$

$$D^3\bar{v}_1 + (\lambda_1 + \sigma)D^2\bar{v}_1 + \sigma\lambda_1 D\bar{v}_1 = k(\sigma - \lambda_1)D\bar{u}_1 \tag{6}$$

in which

$$\lambda_1 = [(\bar{v}_{0\infty}{}^2 + 4\sigma^2)^{1/2} - \bar{v}_{0\infty}]/2$$

and

$$k = 2\sigma^2 G^2 / \{(\lambda_1{}^2 - \sigma^2)(2\lambda_1 + \bar{v}_{0\infty})\}$$

$\bar{v}_{0\infty}$ denotes the value of $\bar{v}_0$ for $\eta \to \infty$.

Results and Discussion

The instability problem is now reduced to find the Görtler parameter G as an eigenvalue together with two components $\bar{u}_1(\eta)$ and $\bar{v}_1(\eta)$ of the perturbation velocities as eigenfunctions. The system of the differential Eqs. (2) and (3) was numerically solved by the method of successive approximation. In Fig. 1 the solid lines show neutral stability curves in relation of the parameter G to the dimensionless wavenumber σ of the longitudinal vortices for several values of the suction parameter C. The stable range is below each neutral stability curve. The curve for $C = 0$ is compared with the previous results in the case without suction (shown as the broken lines *a*, *b*, and *c*). The curve *a* was first calculated by Görtler,[1] the curve *b* by Hämmerlin[2] with higher approximation but with an assumption $v_0 = 0$, and the curve *c* with Galerkin's method by Smith,[4] who considered the streamwise growth of laminar boundary layer. It is found that the boundary layer is less stable when the normal component v_0 is considered. This effect appears to be more remarkable than that[5] for Tollmien-Schlichting instability of a laminar boundary layer along a flat plate. The minimum value of G is found in the case with suction ($C \gtrsim 1.4639$). The value of the Görtler parameter increases as the suction parameter C is increased, and the laminar boundary layer is stabilized. The curve *d* is the neutral curve[6,7] for the so-called asymptotic suction profile $\bar{u}_0 = 1 - \exp(-0.5y/\theta)$. As expected, the present result tends to the curve *d* when $C \to \infty$. Blowing ($C < 0$) gives little effect on the G-value in the state of neutral stability.

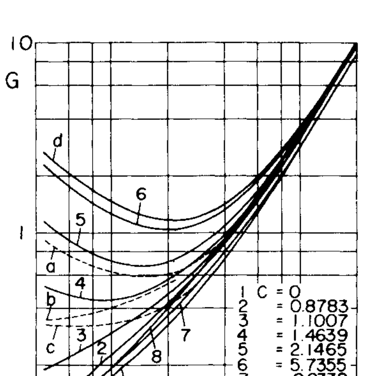

Fig. 1 Neutral stability curves in relation between Görtler parameter *G* and dimensionless wavenumber σ of longitudinal vortices. Broken lines, in the case without suction, are curve *a*: Görtler,[1] curve *b*: Hämmerlin,[2] and curve *c*: Smith.[4] Solid lines, in the case with suction, are ($C > 0$) or blowing ($C < 0$); curve *d*: Kobayashi[6,7] for asymptotic suction profile.

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Impact of Space Shuttle Orbiter Re-Entry on Mesospheric NO_x

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I. Introduction

THEORETICAL modeling of the behavior of minor constituents in the mesosphere is difficult enough even in an assumed steady state but nevertheless, we have performed some time-dependent calculations of odd nitrogen species. The reason we did this was to quantitatively study a situation which has been discussed qualitatively in many circles in the past year or two, namely, what will happen when the proposed space shuttle orbiter re-enters the Earth's atmosphere. The orbiter itself is to be an aircraft 36 m long and 23 m wingspan, weighing 90,000 kg, close to the dimensions of a commercial airliner. Its re-entry is to be characterized by the orbiter's losing most of its orbital kinetic energy along an almost constant altitude leg of its trajectory at about 70 km, extending roughly a quarter of the way round the Earth.

In the shock-heated wake the ambient atmosphere is disturbed; much of the molecular oxygen and nitrogen is converted to nitric oxide, atomic nitrogen, and atomic oxygen. Detailed aerodynamical estimates of the extent of this conversion have been made by Park.[1] Such estimates depend on details of the trajectory, the aircraft materials, and many other aerodynamic considerations. Park has outlined these and shown that for

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